L-H transitions driven by ion heating in scrape-off layer turbulence (SOLT) model simulations

D.A. Russell, D.A. D’Ippolito and J.R. Myra
Lodestar Research Corporation, Boulder, CO, USA

Presented at the
2015 Joint US/EU Transport Task Force Workshop
Salem MA
April 28 - May 1 2015

Work supported by the U.S. Department of Energy Office of Science, Office of Fusion Energy Sciences, under Award Number DE-FG02-97ER54392.
SOLT model equations

- self-consistent evolution of ion pressure and ion diamagnetic drift

Turbulent flow energetics

- generalized Reynolds work ($T_i > 0$, non-Boussinesq)

Source-driven turbulence

- Configurations of density and pressure sources
- L, H and Avalanche (A) regimes visited with increasing $S_{pi}(x,t) \sim t$
- L, H and A regimes of stationary turbulence for time-independent sources.

Conclusions
The SOLT code now includes the self-consistent evolution of ion pressure and ion diamagnetic drift.

- Generalized vorticity is evolved; the Boussinesq approximation has been dropped.
- The new equations of evolution are consistent with the drift-ordered, reduced-Braginskii fluid equations derived by Simakov and Catto* and used in the BOUT code.**
- The electro-static potential is extracted from the vorticity by different algorithms depending on the problem: relaxation on Poisson***, conjugate gradient and multigrid.

**SOLT Model Equations**

Generalized Vorticity ($\rho$)

$$\rho + \nabla \cdot (n\nabla \phi + \nabla p_i) = 0$$

$$\left( \partial_t + v_E \cdot \nabla \right) \rho = -2b \times \kappa \cdot \nabla (p_e + p_i) - J_{||} + \mu \nabla^2 \rho - v_{\parallel} \rho +$$

$$+ \frac{1}{2} \left[ n v_{di} \cdot \nabla \nabla^2 \phi \right] - \frac{1}{2} \left[ v_E \cdot \nabla (\nabla^2 p_i) - \nabla^2 (v_E \cdot \nabla p_i) \right] - \frac{1}{2} b \times \nabla n \cdot \nabla v_E^2$$

$$p_{e,i} = n T_{e,i}, \ v_e = b \times \nabla \phi, \text{ and } v_{di} = b \times \nabla p_i / n \text{ is the ion diamagnetic drift}$$

---

SOLT Model Equations (cont.)

Density (quasi-neutral) \( (\partial_t + v_E \cdot \nabla) n = J_{\parallel,n} + D_n \nabla^2 n + S_n \)

Electron Temperature \( (\partial_t + v_E \cdot \nabla) T_e = q_{\parallel,e} / n + D_{Te} \nabla^2 T_e + S_{Te} \)

Ion Temperature \( (\partial_t + v_E \cdot \nabla) T_i = q_{\parallel,i} / n + D_{Ti} \nabla^2 T_i + S_{Ti} \)

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + v_E \cdot \nabla, \quad v_E = \mathbf{b} \times \nabla \phi \quad (\mathbf{b} \cdot \nabla \times v_E = \nabla^2 \phi), \quad \nabla = \nabla_\perp
\]

\( J_{\parallel} \) models electron drift waves on the closed field lines and sheath physics, through closure relations, in the SOL. \( q_{\parallel} \) models heat flux in the SOL.

All fields are turbulent: \( n = n(x,y,t) \), etc.
We do not expand about ambient profiles in SOLT.
Self-consistent O(1) fluctuations are supported.

Mean Flow production by Reynolds work generalized for Ti > 0 and non-Boussinesq dynamics

\[ g = n \left( v_E + v_{di} \right) = n \mathbf{u} : \text{momentum density} \]

(1) \[ \partial_t g + \nabla \cdot (v_E g) = 0 \Rightarrow \text{vorticity equation} \] (conservative form).

Combine this equation with the density continuity equation to find

(2) \[ \partial_t \mathbf{u} + v_E \cdot \nabla \mathbf{u} = -\mathbf{u} S_n / n \], where \( S_n \) is a source of zero-momentum particles.

\textbf{Mean Flow Energy} \( \varepsilon_{mf} \equiv \frac{1}{2} \langle \mathbf{u} \rangle \cdot \langle \mathbf{g} \rangle \), \( \langle \mathbf{u} \rangle \equiv \text{y-average(} \mathbf{u} \text{)} \)

\[ \partial_t \varepsilon_{mf} + \partial_x q_{mf} = P_{mf} - S_{mf} \]

\[ q_{mf} = \frac{1}{2} \left[ \langle v_x \mathbf{u} \rangle \cdot \langle \mathbf{g} \rangle + \langle v_x \mathbf{g} \rangle \cdot \langle \mathbf{u} \rangle \right] : \text{Mean Flow Energy Flux} \]

\[ P_{mf} = \frac{1}{2} \left[ \langle v_x \mathbf{u} \rangle \cdot \partial_x \langle \mathbf{g} \rangle + \langle v_x \mathbf{g} \rangle \cdot \partial_x \langle \mathbf{u} \rangle \right] : \text{Mean Flow Production} \]

\[ S_{mf} = \left\langle \frac{S_n}{2n} \mathbf{u} \right\rangle \cdot \langle \mathbf{g} \rangle : \text{Energy Loss} \]

\textbf{Reynolds work} : \( P_{mf} - \partial_x q_{mf} \)
Fluctuation Energy: \( \varepsilon_{fl} = \frac{1}{2} \langle u \cdot g \rangle - \varepsilon_{mf} = \varepsilon - \varepsilon_{mf} \)

\[ \partial_t \varepsilon_{fl} + \partial_x q_{fl} = -P_{mf} - S_{fl}, \text{ where} \]

\[ q_{fl} = \frac{1}{2} \langle v_x u \cdot g \rangle - q_{mf} = q - q_{mf} \quad \text{and} \quad S_{fl} = \left\langle \frac{S_n}{2n} u \cdot g \right\rangle - S_{mf} = S - S_{mf}. \]

The total energy is conserved: \( \partial_t \varepsilon + \partial_x q = -S \)

\( P_{mf} > 0 \Rightarrow \) energy transfer from fluctuations to mean flow

\( P_{mf} < 0 \Rightarrow \) turbulence production

\[ T_i = 0 \text{ and Boussinesq approximation (} \nabla n = 0 \text{)} \]

\[ P_{mf} \rightarrow n \cdot \partial_x \left\langle v_{Ey} \right\rangle \cdot \left\langle \delta v_{Ex} \delta v_{Ey} \right\rangle \]

\[ q_{mf} \rightarrow n \cdot \left\langle v_{Ey} \right\rangle \cdot \left\langle \delta v_{Ex} \delta v_{Ey} \right\rangle \]

In the present simulations, these limiting forms are poor approximations to the full expressions.
Particle and energy fluxes are driven by **diffused** \( (D) \) **Gaussian sources** \( (S) \) localized near the core-side boundary. This injection region is well removed from the separatrix \( (\Delta x = 0) \) in the simulations to observe L-H transition phenomena free from SOL physics.

**Stationary Sources**

or

\[
S_{Pi} \sim t
\]

**Confinement Times**

\[
\tau_n = \int_{\Delta x<0} dx \frac{\langle n \rangle}{\int_{\Delta x<0} dx S_n} \\
\tau_P = \int_{\Delta x<0} dx \frac{\langle P \rangle}{\int_{\Delta x<0} dx S_P}
\]
$S_{Pi} \sim t$ : visiting three confinement regimes

L : low confinement times and mean field energy
H : rising confinement times and a broad peak in the history of mean flow energy
A (avalanche) : diminished mean flow energy and bursts in the fluctuation energy

Reynolds Work – global picture
• positive in the L- and H-regimes
• decreasing, with negative bursts in the A-regime
In response to mounting ion pressure, the mean-flow bloom detaches from the source region, initiating the L-H transition.

At the moving front, the mean flow production rate \( P_{mf} \) balances the turbulence injection rate \( \gamma_{mhd} \varepsilon_{fl} \): a moving “Reynolds trigger.”
The front (a) drives a shear layer (b) and leaves a wake of increasing pressure gradient (c) and reduced fluctuation energy.

Avalanches curtail the pressure rise in the wake.
A poloidal array of coherent structures underlies the production front.

- Radial correlation lengths inside the separatrix are long in L and reduced in H.
- Coherent structures are broken up in the A regime.
- The H regime represents a sweet-spot for the location of this phalanx.
The near-source picture supports changes in transport seen near the separatrix.

\[ S_{Pi} \sim t : \text{local Reynolds production} \]

\[ P_{mf}/\gamma_{mhd} \cdot \epsilon_{fl} \]
\[ \Delta x = -7.5 \text{ cm} \]

\[ \langle n \cdot v_{Ex} \rangle \]
\[ \Delta x = -1. \text{ cm} \]

\[ H : \epsilon_{fl} \text{ and particle flux are reduced.} \]

\[ L \text{ and } A \text{ are both low-confinement regimes, but} \]
- \[ L : P_{mf} > 0 \Rightarrow \text{mean flow production by turbulence} \]
- \[ A : P_{mf} < 0 \Rightarrow \text{turbulence production by mean flow (K-H?)} \]
Confinement times reveal distinct regimes similar to those seen in the $S_{Pi} \sim t$ study.

$H$: confinement times increase with increasing ion heating ($S_{Pi}$).

$L, A$: confinement times decrease with increasing ion heating.
• A high-shear layer provides a transport barrier (circled) in the H-mode: \(|v'_E| > \gamma_{\text{mhd}}\).
• This barrier is absent from the L-mode.

Local diagnosis can misguide global prediction.

What you see depends on where you look.

• Global nonlinear analysis \((P_{\text{mf}}, \varepsilon_{\text{mf}})\) is in progress.
We find 3 different confinement regimes with increasing ion heating ($S_{\text{Pi}}$) in a 2D source-driven fluid turbulence model that retains the ion diamagnetic and gyroviscous effects.

- The regimes L, H and A are reminiscent of tokamak L, H, and ELMy-H mode regimes.
- Enhanced confinement in the H regime is associated with the movement of a shear layer to just inside the separatrix; $|v'_E| > \gamma_{\text{mhd}}$ in the layer.
- Our model does not have sufficient physics to describe peeling-balloonning ELMs, but rather A likely involves the K-H instability.
- The relationships between $v'_E$, $v'_\text{di}$ and pressure gradient ($\gamma_{\text{mhd}}$) depend strongly on radial location, making local diagnosis ambiguous.
- A global energetics model, taking into account ExB and diamagnetic flows, has been developed and is being applied to the simulations.