Interaction of Resonant Magnetic Perturbations and Neo-Classical Tearing Modes with Turbulence

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Outline

• Introduction
• Quiescent islands (with anomalous transport coefficients)
• Turbulent islands
  – Fluid description
  – Gyrokinetic description
• Discussion
Islands modify profiles in DIII-D…

Nazikian et al, PRL 2015

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... as well as LHD and JT-60

Ida et al., NF 2004

Ida et al., PRL 2012
Goals of island theory

• There are two main goals to island theory:
  1. Predict island size. Two cases must be distinguished:
     a. Driven island (RMP)
     b. Intrinsic island (TM)
  2. Predict profile modifications for a given island size

• The two questions are inseparable
General theory of magnetic islands
Island evolution is determined by two generalized Rutherford equations

- The island width $W$ and its propagation velocity $V$ are governed by:
  \[
  \frac{dW}{dt} = \Delta'(W,V) + \Delta(W, V); \\
  \frac{dV}{dt} = F(W, V) = 0.
  \]
- $\Delta'$ measures the drive for reconnection (island growth) from currents outside the island, including plasma, wall & RMP.
- $\Delta$ & $F$ represent, respectively, the local drive and the forces acting on the island in the binormal direction.
- Classical tearing modes occur when $\Delta'>0$.
- Neoclassical Tearing Modes occur when $\Delta>-\Delta'$. 
These equations have their origin in exact conservation laws

- The amplitude equation comes from the flux-surface average of Ohm’s law:
  \[
  \langle \partial_t A \rangle + \langle b \cdot \nabla \cdot \Pi \rangle = \eta \langle J \rangle
  \]

- The phase velocity equation comes from the flux-surface average of the charge conservation equation
  \[
  \partial_t \langle V_y \rangle + \langle V_r U_\parallel \rangle = \langle (\nabla \cdot \Pi)_y \rangle + \langle B_r J_\parallel \rangle + \mu \langle \nabla^2 V_y \rangle
  \]

- Full, closed system must be solved
Magnetic islands are like a sailboat

- Electrons and ions = wind and water
- The tearing drive $\Delta = \text{vertical forces}$ (buoyancy + hydro lift), while the binormal force $= \text{the sum of the lateral forces on sails and keel.}$
- The sails push the boat up its bow wave
- With enough wind the boat can climb on top of its bow wave and surf ($= \text{island suppression}$)
The Reynolds stress (ion inertia) gives rise to a polarization current

- Analytic calculations for large islands find
  \[ \Delta_{\text{pol}}(W,\omega) = g \ V \ (V - V_{*i})/w^3 \]
- Quadratic dependence on velocity is explained by proportionality to product of flux (\(\sim V\)) and captured momentum (\(\sim V - V_{*i}\)), as in Newton’s theory for lift.
- The challenge is to determine \( V \).
The “lift” and “drag” can be observed

- The equations of motion for the island are
  \[ \frac{dW}{dt} = \Delta' - \Delta(W, V) \]
  \[ \frac{dV}{dt} = F(W, V) \]

- The growth rate for the island is thus a measure of the lift

- The acceleration of the island when the island velocity is swept gives a measure of the binormal force

Zohm, NF 2001
Electrostatic simulations allow the island response to be charted

- Like a channel or wind tunnel measurement, they allow direct calculation of D and F as functions of W and V
- At constant field strength, unlocking occurs when the channel velocity increases above a threshold, here $0.05 v_A$ (Militello, NF 2009)
Quiescent islands

Turbulence modeled through anomalous transport coefficients
Simple model predicts island velocity

- **Frozen-in assumption:**
  - electrons are trapped inside separatrix
  \[ u_{isl} = v_{E}^{in} + f v_{De}, \]
  where \( f \) is the flattening fraction.

- **No-slip:**
  - ion velocity is continuous across separatrix,
  \[ f v_{Di} + v_{E}^{in} = v_{Di} + v_{E}^{out} \]

- It follows that
  \[ u_{isl} - v_{E}^{out} = f v_{De} + (1-f) v_{Di} \]
1. Profile change in dragged islands

- For cold ions
  \[ u_{isl} - v_E^{\text{out}} = f v_{De} \]

- If \( u_{isl} - v_E^{\text{out}} > v_{De} \), \( f > 1 \): the density profile must steepen.

- If \( (u_{isl} - v_E^{\text{out}}) / v_{De} < 0 \), \( f < 0 \): the density profile must reverse.

Yu & Günter, NF 2009
2. Discontinuity in $E_r$ causes large destabilizing $J_{pol}$ for $W>4\rho_i$

- Wake excitation invalidates the no-slip assumption.
- Steep velocity gradients around the separatrix yield strong polarization currents

Finite-amplitude destabilization occurs even without bootstrap current

- We fit the results with
  \[
  \Delta_{pol} = g \frac{(V - V_1)(V - V_2)}{W(W_c^2 + W^2)}
  \]
- This is an interpolation between large and small W limits
- The limits of the stability band decrease faster than the island velocity as the island grows.
- The wake created by the island is visible in plots of the current density
2D Fluid Simulations of turbulent islands

Transport time scale
2D electrostatic simulations shed light on earlier electromagnetic study

- Electromagnetic simulations of ITG turbulence (Ishizawa & Nakajima 2010) exhibit an inverse cascade of energy from short to long wavelength magnetic fluctuations.
- This cascade can be understood either as resulting from island coalescence or from turbulence producing a direct drive on long wavelength island.
- The island acts as a “zonal field” for the turbulence: it both draws energy from the turbulence and affects ITG stability.
Turbulence shifts force curves

- The turbulence enhances the viscous force acting on the island.
- It enhances the polarization current directly as well as by shifting the natural frequency.
- Note that island coalescence does not occur in ES model.
Free island experiences “zonal” drive

- Anomalous viscosity is about $0.2 \rho_s^2 v_{Ti} / L$.
- The drive is dominated by the contribution from the small scales, with a weaker contribution from the coherent polarization current

$$W / \rho_s = 3.5; \quad \eta_i = 2.5; \quad F_y(u_{free}) = 0;$$
GYRO simulations of turbulent islands

Long compared to turbulence saturation scale but short compared to confinement time
Island profile modifications are similar to fluid predictions

$\Delta n=1$ GA-std case: $E_r^0 = +8 \& 0$, $\Delta A^\text{ext} = -0.006$ ($w/a = 0.05$), $\hat{\gamma}_E = 0.1$, $\hat{\gamma}_\phi = 1.2$

- expected island grad-$T_e$ flattening inside & steepening outside independent of $E_r$, limited by trapping (cf Park & Chang)
- grad-$n$ close to adia-e
- $-\partial \Delta n/\partial r \sim n(e/T_e) \partial \Delta \phi/\partial r$
- The lowering of $[a/L_{Ti}]_{\text{ave}} = 3 \Rightarrow 2.5$ is a problem.
EM Gyrokinetic simulations also show unlocking and suppression

- Island initially shifts its phase to achieve force balance.
- Further phase slippage leads to unlocking and healing.
- Final state is a screened island with the response field canceling the vacuum field at the resonant surface.

Waltz and Waelbroeck, PoP 2012.
EM GYRO drive is qualitatively consistent with quiescent results

- Analytic calculations for \( w \gg \rho_i \) (Fitzpatrick and Waelbroeck PoP ‘06) find
  
  \[
  D_{\text{pol}}(w,v) = \frac{[a+bvw+c(vw)^2]}{w^3}
  \]

- Quadratic dependence on velocity recovered, as in Newton’s theory for lift.
- Stable band of propagation velocities is apparent.
Tearing mode theory and observations have made progress

- Theory and simulations of the polarization drive are in good agreement for all $W, V$.
- There is a discrepancy in the predictions of the free propagation velocity $V$ for medium islands ($\omega_0 \sim k \parallel c_s$) but a probable cause (the wake) has been identified.
- Theory (Heyn ‘08) predicted the island at the top of the pedestal in suppressed-ELM discharges.
- Our growing understanding of profile modification may lead to new methods for RT control of islands.
Knowledge from turbulent & NC transport will drive future progress

• There remains uncertainty over the role of the polarization current in setting the NTM threshold.

• Our understanding of mode penetration is inadequate (why almost always q=2? Why linear scaling with density?)

• Gaining confidence in predicting island widths is key for the future of stellarators, especially the QS variety.

• New 3D simulations and new observations using synchronous analysis of rotating islands (Z. Taylor, R LaHaye et al.) yield unprecedented information.