Full-wave Electromagnetic Field Simulations in the Lower Hybrid Range of Frequencies

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Outline

• **Background and motivation**
  – Breakdown of geometrical optics approximation in the LHRF regime
  – Feasibility of full-wave LHRF simulations on massively parallel platforms - resolution requirements

• **Adaptation of full-wave solver to MPP device**
  – Matrix inversion algorithm

• **LHRF simulations**
  – Convergence study
  – Observation of spectral broadening at LH wave caustics
Wave propagation in LHRF regime is usually treated using toroidal ray tracing

- Integrate the ray equations of geometrical optics:

\[
\frac{d\mathbf{x}}{dt} = -\frac{\partial \varepsilon}{\partial \mathbf{k}} \quad \frac{d\mathbf{k}}{dt} = \frac{\partial \varepsilon}{\partial \omega}
\]

- This method treats 2-D equilibrium effects \([\mathbf{B}(r,\theta)]\) on \((k_\parallel, k_\perp)\) evolution of LH wave:

\[
k_\parallel = \mathbf{k} \cdot \mathbf{b} = \frac{m}{r} b_\theta + \frac{n}{R} b_\phi \quad \mathbf{b} = \frac{\mathbf{B}}{|\mathbf{B}|}
\]

- Approach is valid in short wavelength limit \(\Rightarrow \lambda_\perp << L_p\)
Caustic Formation is a Ubiquitous Feature of LH Ray Trajectories in Tokamaks

- Geometrical optics breaks down at caustics where the initial wavefront focuses down to a scalelength of the order of \( \lambda_\perp \)

- At caustics, geometrical optics does not account for full-wave effects such as diffractional broadening of the LH wavefront [G. Pereverzev NF, 1992].
Full-wave Formulation and Spectral Ansatz

Solve Maxwell's equations for a fixed frequency with a linear plasma response in a mixed spectral-finite element basis using the TORIC EM field solver [Brambilla, Wright, Bonoli]:

\[ \nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \left[ \mathbf{E} + \frac{4\pi i}{\omega} \left( \mathbf{J}^p + \mathbf{J}^{\text{ANT}} \right) \right] \]

\[ \mathbf{J}^p = \overline{\sigma} \cdot \mathbf{E} \]

\[ \mathbf{E}(\mathbf{x}) = \sum_m \mathbf{E}_m(\psi)e^{i(m\theta+n\phi)} \]

- The antenna is modeled as a current sheet, \( J_A(\psi_A,\theta,n) \):
Conductivity Relation - LHRF Regime

- LHRF Regime:
  - Conductivity relation evaluated in the limit of $\Omega_{ci}^2 << \omega^2 << \Omega_{ce}^2$ and $\omega \geq 2\omega_{LH}$:
  - Unmagnetized ions
  - Strongly magnetized electrons $[(k_\perp \rho_e)^2 << 1]$
  - Wave equation is sixth order with two propagating modes:
    - Electrostatic LH “slow wave” branch - damps via ELD
    - Electromagnetic LH “fast wave” branch - damps via ELD and electron TTMP
    - Mode converted ion plasma wave is not propagative
Lower Hybrid Dispersion Relation

- Frequency range is intermediate of ion and electron cyclotron frequencies: \( \Omega_{ci}^2 \ll \omega^2 \ll \Omega_{ce}^2 \)

- The cold plasma Dispersion Relation (DR) describes two propagating waves ("slow" and "fast" modes)

\[
P_4 n_\perp^4 + P_2 n_\perp^2 + P_0 = 0
\]

Slow mode is electrostatic (larger \( n_\perp \)) and couples by \( E_\parallel \)
Fast mode is electrostatic (smaller \( n_\perp \)) and couples by \( E_\perp \)

- Predicts an accessibility criterion:

\[
\pi_\parallel > \pi_a \equiv \frac{\omega_{pe0}}{\Omega_{ce0}} + \varepsilon_{\perp0}^{1/2}
\]
Requirements to resolve the “slow” electrostatic LH wave in Alcator C-Mod

\[ k_\perp^2 \approx -\frac{\varepsilon_\parallel}{\varepsilon_\perp} k_\parallel^2 \]

\[ \varepsilon_\perp = 1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2} \approx 1 \]

\[ \varepsilon_\parallel = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \approx -\frac{\omega_{pe}^2}{\omega^2} \]

\[ \Rightarrow k_\perp \approx \frac{\omega_{pe}}{\omega} k_\parallel \]

\[ B_0 = 4.5 \text{ T}, \quad D^+ \]

\[ f_0 = 4.6 \text{ GHz} \quad n_\parallel = 2.5 \]

\[ n_{e0} = 2 \times 10^{20} m^{-3} \]

\[ \omega / \Omega_{cD} \approx 135 \]

\[ k_\perp \approx 66 \text{ cm}^{-1} (\lambda_\perp \approx 1 \text{ mm}) \]
Small wavelength requires high spectral resolution

• For mode converted IBW and ICW ⇒ $k_\perp \rho_i \approx 1$:
  $k_\perp \approx (m/r)$ then $M_{\text{max}} \approx 255$ for typical tokamak parameters.

• For LH waves ⇒ $k_\perp \approx (\omega_{pe}/\omega)k_\parallel$:
  $k_\perp \approx (m/r)$ ⇒ $M_{\text{max}} \approx 1000$

• 4-fold increase in poloidal resolution requires 16 times more memory and 64 times more computation
  -> parallelization required
MPP Implementation of TORIC Solver

• Adapted an out of core Thomas solver to perform matrix inversion.

• The TORIC algorithm results in a block tridiagonal stiffness matrix, where each block is \((6*N_m)^2\), with \(3*N_r\) blocks.

• These blocks can be of the order of 1GB, and so, (SCA)LAPack is used to do an LU decomposition, with the memory of each block distributed across the processors.
  – Partial decomposition stored on local hard disks of nodes

• Simulations performed on 50 processor MIT Beowulf cluster
Prescription for coupling to the slow wave: Launch a fast wave with $n_{\parallel}$ below $n_a$.

The slow wave accessibility criterion is met several centimeters into the plasma.

At that point, the fast and slow waves coalesce and the launched fast wave can mode convert to the slow wave.

Plasma Parameters - Alcator C-Mod

- $D(5\% \text{ H})$, $f_0 = 4.6 \text{ GHz}$
- $B_0 = 5.3 \text{ T}$, $I = 1 \text{ MA}$
- $T_e = 3.5 \text{ keV}, T_i = 2.0 \text{ keV}$
- $n_e(0) = 1.5 \times 10^{20} \text{ m}^{-3}$
- $n_{\parallel} = 1.5$, $n_a(0) = 2.0$
Full-wave results with $N_m = 255$ and $N_r = 480$ are not yet converged.
Simulations finally converge with $N_m = 1023$ and $N_r = 960$

- The FW is coupled at the edge, then mode converts to a slow wave at the confluence.
- The slow wave propagates out to the edge cutoff and reflects inward again to the confluence.
- Loci of edge reflections and mode conversions in the full wave field pattern forms a “ring” of accessible LH waves.
- Simulation required 8064 MPP’s on Beowulf cluster (7 day utilizing 48 nodes)
Simulation describes “Ring” of LH Waves

- All wave power is absorbed on the electrons by Landau damping.
- Caustic formation is apparent.
- There is also a suggestion of resonance cones in the full wave field patterns.
Caustic formation in full-wave fields follows the ray tracing prediction
Spectral shift is large at caustic

• The distribution of $n_\parallel$ on flux surfaces shows a significant upshift from an averaged launched $n_\parallel$ of 2 to >4 in the middle of the annulus at $r/a=0.75$.

• This rapid upshift at the caustic causes all the power to be absorbed in the narrow region bound by the caustic and the edge cutoff.
The local $n_{||}$ evolves to 2.5 on the high field side from 1.5 at the antenna.

$N_{||}$ evolution is due primarily to geometric effects of major radius position, and results in different damping.

Rays penetrate farther into the plasma and damp, whereas damping in the full-wave simulation is confined to an annulus.
Power deposition profiles are both localized, but at different radii

**TORIC**

- TORIC shows power deposition is confined to a narrow region of \( r/a = [0.65, 0.83] \)

**ACCOME**

- ACCOME shows damping penetrating to the center and starting within the 'caustic'.

- The difference may be related to the downshifting of \( v_\phi \) by diffraction, which is high at the caustic - evidence for full wave enhancement in \( k_{//} \), filling the spectral gap.
Summary

• Use of an MPP device has permitted LHRF full-wave studies in toroidal geometry for the first time ever.

• Important full-wave phenomena including focussing and diffraction can now be accurately assessed:
  – Significant spectral broadening is found near LH wave caustics.
  – The spectral broadening of $k_{//}$ observed in our simulations is sufficient to bridge the spectral gap in LH current drive between injected waves at $(5-8)v_{te}$ and the phase speeds necessary for electron damping at $(2.5-3)v_{te}$.

• More simulations are needed at higher spectral resolution ($N_m \approx 2000$) to test the ray tracing result that some LH power does penetrate to the plasma core.